Study the system by two different method, Markov-renewal processes and Laplace transforms techniques.

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Abstract

This paper studies two-unit standby system where each unit has two different modes, (normal, total failure). We study the system by two different methods to find over upgrade method to solve any system more easy. First method by using Markov-renewal processes and I solved it later. The second method by using Laplace transforms techniques. The failure and repair time of each unit are taken as exponential distribution in two methods.

Keywords: Availability, Reliability, Cold standby, Laplace transforms, supplementary technique, Markov-renewal processes.

1. Introduction

Redundancy techniques are widely used to improve system performance in terms of reliability and availability. Many authors study stochastic model of a two- unit cold standby redundant system subjected to random failure. The random failure occurs at random times, which follow exponential distributions. [1] The purpose of this paper is to study the steady-state availability of a repairable system with standby switching failure. The repairable system configuration includes the primary and standby components, where an unreliable server is responsible to repair or monitor the failed ones. The time-to-failure and time-to-repair of the components follow exponential and general distribution, respectively. The server subjects to active breakdown when it is repairing. The time-to-breakdown of the server is also assumed to be exponentially distributed. When the primary components fail, the standby components replace the primary components successfully with probability $1-q$. The repair time of the failed components and the repair time of the breakdown server are generally distributed. Further, we frame a practical model with three different repairable system configurations. We use supplementary variable method and integro-differential equations to obtain the steady-state availability of these three different repairable system configurations. [2] This investigation is concerned with an M/G/1 machine interference problem with imperfect switchover of standbys, in which an unreliable repairman maintains a group of machines. An unreliable repairman means that the repairman is typically subject to unpredictable breakdowns. The time between two consecutive breakdowns follows an exponential distribution, and recovery time of the unreliable repairman follows a general distribution. The lifetime of operating/standby machines and the repair time of failed machines are exponentially and generally distributed, respectively. Using the method of supplementary variable, the stationary probability distribution is obtained. We develop some performance measures and system reliability indices. Furthermore, the cost effectiveness maximization is also discussed. Finally, a cost model is proposed to find the optimal numbers of operating and standby substations, which minimize the average cost per unit time. [5] discussed a system model with server failure and unit replacement at different failure modes, followed by inspection but they completely ignored the importance of redundancy as is needed in many critical systems like nuclear power plants, remote sensing, communication satellites etc. Moreover, in many studies, such as. [4], and [3], the failure and repair times of units are assumed to follow, ridiculously, the exponential distribution. But it is not reasonable

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for each and every situation. Rather, we should use suitable distribution in accordance with prevailing situation

This paper is devoted to deal with a two-unit standby system where each unit has two different modes, (normal & total failure) and two phases of repair. We deal with one system solved by two methods. First method by using Markov-renewal processes and the second method by using Laplace transform techniques

1.1. Notations

λ_1 Failure rate from O to F1 , λ_2 Transition rate from F1 to F2

μ_1 Repair rate from F1 to O , μ_2 repair rate from F2 to O

$p_i(t)$ Probability for i=0, 1, 2, 3 , $p_i^*(s)$ Laplace transform of $p_i(t)$

A(t): availability function of the system. , R (t): reliability function of the system.

MTTF: mean time to system failure.

Laplace transform of $p_i(t)$ are defined as:

$p_i^*(s) = \int_0^\infty e^{-St}\, p_i(t)\, dt$, i=0,1,2,3

1.2. Model description

The following assumptions are common for the system

1- System consists of two identical units, one unit is operative and the other is in cold standby.

2- Upon failure of the operative unit the standby unit becomes operative if it is not failed

3-The system fails completely if during the repair of the failed unit, the other unit also fails.

4. Two repairmen are available; the first is to repair in phase I while the second is to repair in phase II. After repair, the unit becomes as good as new.

- **A unit can be in one of the following states**

O unit operative , S unit cold standby

F_1 Failed unit is under repair in phase I , F_2 failed unit is under repair in phase II.

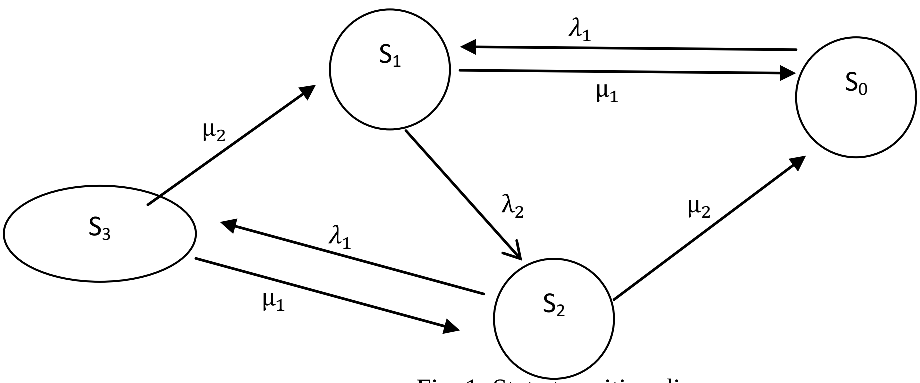

Fig. 1: State transition diagram

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Considering these symbols, the system may be in one of the following states

Up state: So = (O, S) , S_1 = (F_1, O) , S_2= (O, F_2,),
Down state: S_3 = (F_2, F_1)

First method (by using Markov-renewal processes)

1.3. Mathematical model description

Figure (1) gives the state Transition diagram for the warm standby system. We see that S3 is an absorbing state. We obtain the following differential equations: (adapted from Wang & Liou, 2006).

$$\frac{dP_0(t)}{dt} = -\lambda_1 P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) \quad (1)$$

$$\frac{dP_1(t)}{dt} = \lambda_1 P_0(t) - [\mu_1 + \lambda_2] P_1(t) \quad (2)$$

$$\frac{dP_2(t)}{dt} = -[\mu_2 + \lambda_1] P_2(t) + \lambda_2 P_1(t) \quad (3)$$

$$\frac{dP_3(t)}{dt} = \lambda_1 P_2(t) \quad (4)$$

If we let P (t) denote the probability row vector at time t. then the initial Conditions for this problem

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0)] = [1,0,0,0] \quad (5)$$

This can be written in the matrix form as

$$P^{/} = QP \quad (6)$$

Where,

$$Q = \begin{bmatrix} -\lambda_1 & \mu_1 & \mu_2 & 0 \\ \lambda_1 & -(\mu_1 + \lambda_2) & 0 & 0 \\ 0 & \lambda_2 & -(\mu_2 + \lambda_1) & 0 \\ 0 & 0 & \lambda_1 & 0 \end{bmatrix}$$

And $P = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$

Without deriving the transient solutions, we use the above procedure to develop an explicit expression for the MTTF by taking the transpose matrix of Q and delete the rows and columns for the absorbing states. The new matrix is called A. the expected time-to-reach an absorbing state is calculated from

E [TP(0)→P(absorbing)]= P(0)(- A^{-1})$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and

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P (0) = [1,0,0,0]

Where,

$$A = \begin{bmatrix} -\lambda_1 & \lambda_1 & 0 \\ \mu_1 & -(\mu_1+\lambda_2) & \lambda_2 \\ \mu_2 & 0 & -(\mu_2+\lambda_1) \end{bmatrix}$$

1.4. Reliability Analysis

The mean time to system failure is defined as the time for the system to be completely inoperative

We obtain the following explicit expression for the MTTF:

$$MTTF = \frac{(\mu_2+\lambda_1)(\mu_1+\lambda_2)+\lambda_1(\lambda_1+\lambda_2+\mu_2)}{\lambda_2\lambda_1^2}$$

1.5. Availability Analysis

Calculations for system with warm standby unit: The initial conditions are the same in equation (5). So we obtain the following differential equations:

$$\frac{dP_0(t)}{dt} = -\lambda_1 P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) \quad (7)$$

$$\frac{dP_1(t)}{dt} = \lambda_1 P_0(t) - [\mu_1+\lambda_2]\, P_1(t) + \mu_2 P_3(t) \quad (8)$$

$$\frac{dP_2(t)}{dt} = -[\mu_2+\lambda_1]\, P_2(t) + \lambda_2 P_1(t) + \mu_1 P_3(t) \quad (9)$$

$$\frac{dP_3(t)}{dt} = \lambda_1 P_2(t) - [\mu_1+\mu_2]\, P_3(t) \quad (10)$$

The differential equations form can be expressed as

$$\begin{bmatrix} P_0^l \\ P_1^l \\ P_2^l \\ P_3^l \end{bmatrix} \begin{bmatrix} -\lambda_1 & \mu_1 & \mu_2 & 0 \\ \lambda_1 & -(\mu_1+\lambda_2) & 0 & \mu_2 \\ 0 & \lambda_2 & -(\mu_2+\lambda_1) & \mu_1 \\ 0 & 0 & \lambda_1 & -(\mu_1+\mu_2) \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

The steady state availability of the system is given by

$$A_0(\infty) = 1 - P_3(\infty) \quad (11)$$

And Q $^{P(\infty)}$ = 0

Or, in the matrix form

$$\begin{bmatrix} -\lambda_1 & \mu_1 & \mu_2 & 0 \\ \lambda_1 & -(\mu_1+\lambda_2) & 0 & \mu_2 \\ 0 & \lambda_2 & -(\mu_2+\lambda_1) & \mu_1 \\ 0 & 0 & \lambda_1 & -(\mu_1+\mu_2) \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (12)$$

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To obtain $P_3(\infty)$, we use normalizing condition

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)=1 \tag{13}$$

Then we substitute by (9) in any one of the redundant rows (8).it yields:

$$\begin{bmatrix} -\lambda_1 & \mu_1 & \mu_2 & 0 \\ \lambda_1 & -(\mu_1+\lambda_2) & 0 & \mu_2 \\ 0 & \lambda_2 & -(\mu_2+\lambda_1) & \mu_1 \\ 1 & 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The explicit expression of the steady-state availability is:

$$A_0=\frac{N_2}{D_2} \tag{14}$$

Where,

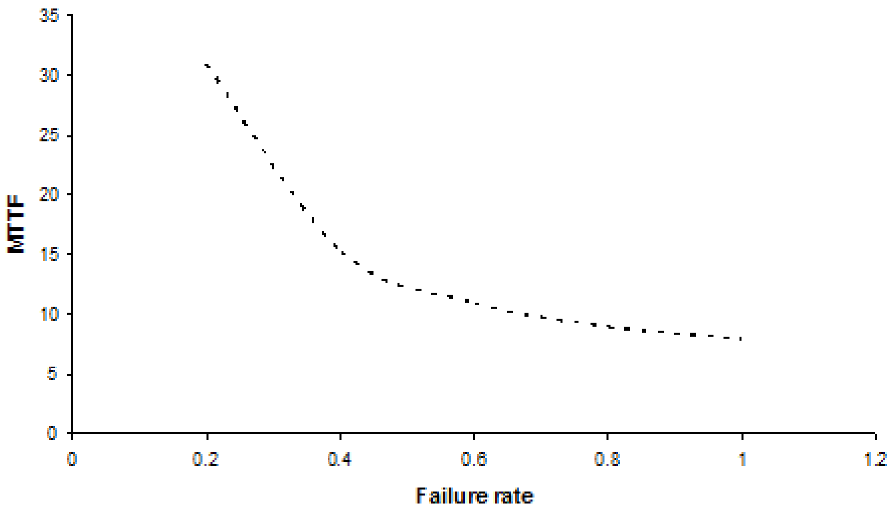
$$N_2=\mu_2[\mu_2(\mu_1+\lambda_1+\lambda_2)+\lambda_1(2\mu_1+\lambda_1+\lambda_2)+\mu_1(\mu_1+\lambda_2)]+\mu_1\lambda_1\lambda_2$$

And

$$D_2=\mu_2[\mu_2(\mu_1+\lambda_1+\lambda_2)+\lambda_1(2\mu_1+\lambda_1+\lambda_2)+\mu_1(\mu_1+\lambda_2)]+\lambda_1\lambda_2(\mu_1+\lambda_1)$$

2. Study the behavior of the system through graphs:

We plot the MTTF and the steady -state availability for the models, against λ_1 keeping the other parameters fixed At $\lambda_2=0.25$, $\mu_1=\mu_2=0.20$,



The MTTF w.r.t. Failure of Component O to f1

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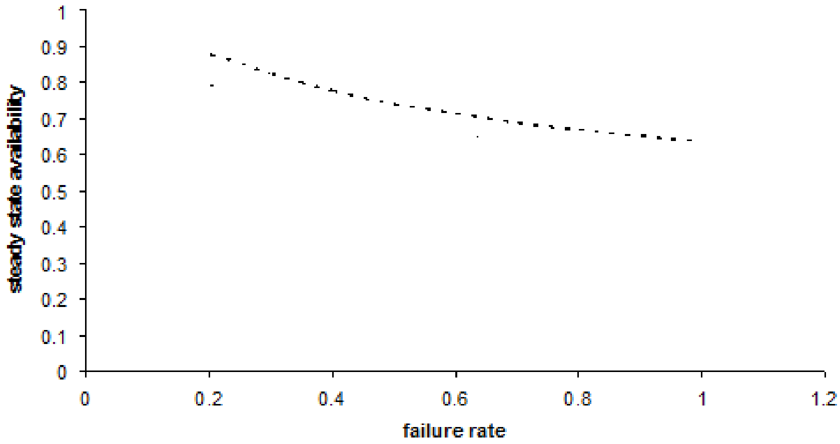

The Steady state Availability w.r.t. Failure Rate of O to f1

Second method (using Laplace transform techniques)

3. Mathematical model description

According to system configuration diagram in fig.1, the difference – differential equations for this stochastic process which is continuous in time and discrete in space are given as follows.

$$\frac{dP_0(t)}{dt} = -\lambda_1 P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) \quad (1)$$

$$\frac{dP_1(t)}{dt} = -[\mu_1 + \lambda_2] P_1(t) + \lambda_1 P_0(t) + \mu_2 P_3(t) \quad (2)$$

$$\frac{dP_2(t)}{dt} = -[\mu_2 + \lambda_1] P_2(t) + \lambda_2 P_1(t) + \mu_1 P_3(t) \quad (3)$$

$$\frac{dP_3(t)}{dt} = -[\mu_1 + \mu_2] P_3(t) + \lambda_1 P_2(t) \quad (4)$$

Initial conditions:

$$P_i(0) = \begin{cases} 1 & \text{where } i = 0 \\ 0 & \text{otherwise} \end{cases}$$

Taking Laplace transform of equations (1) – (4) , we get

$$[\lambda_1 + s] P_0^*(s) - \mu_1 P_1^*(s) - \mu_2 P_2^*(s) = P_0(0) \quad (5)$$

$$[\mu_1 + \lambda_2 + s] P_1^*(s) - \lambda_1 P_0^*(s) - \mu_2 P_3^*(s) = P_1(0) \quad (6)$$

$$[\mu_2 + \lambda_1 + s] P_2^*(s) - \lambda_2 P_1^*(s) - \mu_1 P_3^*(s) = P_2(0) \quad (7)$$

$$[\mu_1 + \mu_2 + s] P_3^*(s) - \lambda_1 P_2^*(s) = P_3(0) \quad (8)$$

Solving equations (5-8) by crammer rule, we obtain:

$$P_0^*(s) = \frac{\Delta 1}{\Delta} \quad , \quad P_1^*(s) = \frac{\Delta 2}{\Delta}$$

$$P_2^*(s) = \frac{\Delta 3}{\Delta} \quad , \quad P_3^*(s) = \frac{\Delta 4}{\Delta}$$

Where,

$$\Delta = s[s^3 + a_1 s^2 + a_2 s + a_3] \quad , \quad \Delta_1 = s^3 + As^2 + Bs + m$$

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$\Delta 2 = \lambda_1 s^2 + (\lambda_1^2 + {}_1\lambda_1 + 2\mu_2\lambda_1)s + {}_2\lambda_1^2 + {}_2^2\lambda_1 + {}_1\ {}_2\lambda_1)$

$\Delta 3 = \lambda_1\lambda_2 + (S + {}_1 + \mu_2)$, $\Delta 4 = \lambda_1^2\lambda_2$

We know that the system contain of (3) up state and (1) down state so

$$P_0^*(s) = \frac{\Delta 1}{\Delta} = \frac{s^3 + As^2 + BS + m}{s[s^3 + a_1 s^2 + a_2 S + a_3]} \quad (6)$$

Where,

$a_1 = 2\lambda_1 + \lambda_2 + 2\mu_1 + 2\mu_2$

$a_2 = \mu_1^2 + 3\mu_1\mu_2 + \mu_1\lambda_2 + 2\mu_1\lambda_1 + {}_2^2 + 3\mu_2\lambda_1 + 2\mu_2\lambda_2 + \lambda_1^2 + 2\lambda_1\lambda_2$

$a_3 = {}_1^2\ {}_2 + \mu_2^2\lambda_2 + {}_1\ {}_2^2 + 2\mu_1\ {}_2\lambda_1 + {}_1\ {}_2\lambda_2 + \mu_1\lambda 1\lambda_2 + {}_2^2\lambda_1 + {}_2\lambda_1^2 + {}_2\lambda_1\lambda_2 + \lambda_1^2\lambda_2$

$m = {}_1\ {}_2^2 + {}_1^2\ {}_2 + {}_1\ {}_2\lambda_1 + \mu_2^2\lambda_2 + {}_1\ {}_2\lambda_2$

$B = 3\mu_1\ {}_2 + {}_1\lambda_1 + {}_1\lambda_2 + {}_2\lambda_1 + 2\mu_2\lambda_2 + \lambda_1\lambda_2 + \mu_1^2 + {}_2^2$

$A = 2\mu_1 + 2\mu_2 + \lambda_1 + \lambda_2$

4. Cubic equations roots have are two cases

After test for (D) we found that D > 0 so we use First case (D > 0)

$$P_0^*(s) = \frac{\Delta 1}{\Delta} = \frac{s^3 + As^2 + BS + m}{s(s + A_1 - W)(S + A_1 + w_1 - i\sqrt{3}v_1)(S + A_1 + w_1 + i\sqrt{3}\,v_1)} \quad (7)$$

By using the inverse of Laplace Transform of equation we obtain

$$P_0(t) = \frac{m}{(A_1 - w)({A_1}^2 + A_1 w + {w_1}^2 + 3{v_1}^2)} + \frac{(-A_1 + w)^3 + A(-A_1 + w)^2 + B(-A_1 + w) + m}{(-A_1 + w)(9{w_1}^2 + 3{v_1}^2)} e^{(-A_1 + W)t}$$

$$+\left\{\frac{2[PX + 3HT][\cos\sqrt{3}(v_1)t] - 2\sqrt{3}\,[(HX) - (TP)](\sin\sqrt{3}(v_1)t)}{X^2 + 3T^2}\right\} e^{(-A_1 - w_1)t} \quad (8)$$

Where,

$P = (-{A_1}^3 - 3{A_1}^2 w_1 + 9A_1{v_1}^2 - {w_1}^3 + 9w_1{v_1}^2 - A_1{w_1}^2) + A({A_1}^2 + A_1 w - 3{v_1}^2 + {w_1}^2) + B(-A_1 - w_1) + m$

$H = (6A_1 v_1 w_1 - 3{v_1}^3 + 3{w_1}^2 v_1 + 3{A_1}^2 v_1 - AA_1 v - A\,v_1 w + Bv_1)$

$X = 3V^2 W + 6{v_1}^2 A_1$, $T = 3Wv_1 A_1 + 3{w_1}^2 V - 6{v_1}^3$

4.1 Reliability System and availability

To obtain the reliability function for this model, we assume that at least one of failed states is absorbing state and the transition rate from this state equal to zero.

4.1.1 System availability

We obtain the availability function for this model, from the following relation

$$A(t) = \frac{m}{(A_1 - w)({A_1}^2 + A_1 w + {w_1}^2 + 3{v_1}^2)} + \frac{(-A_1 + w)^3 + A(-A_1 + w)^2 + B(-A_1 + w) + m}{(-A_1 + w)(9{w_1}^2 + 3{v_1}^2)} e^{(-A_1 + W)t}$$

$$+\left\{\frac{2[PX + 3HT][\cos\sqrt{3}(v_1)t] - 2\sqrt{3}\,[(HX) - (TP)](\sin\sqrt{3}(v_1)t)}{X^2 + 3T^2}\right\} e^{(-A_1 - w_1)t} \quad (9)$$

The steady – state availability can be obtained from the following relation

$$A=\lim_{t\to\infty} A(t) =\frac{m}{(A_1 - w)({A_1}^2+A_1 w + {w_1}^2+3{v_1}^2)} \tag{10}$$

4.1.2. System reliability

To obtain the reliability function for this model, we assume that at least one of failed states is absorbing state and the transition rate from this state equal to zero

$$R(t)= \frac{m}{(A_1 - w)({A_1}^2+A_1 w + {w_1}^2+3{v_1}^2)} + \frac{(-A_1+w)^3+A(-A_1+w)^2+B(-A_1+w)+m}{(-A_1+w)(9{w_1}^2+3{v_1}^2)} e^{(-A_1+W)t}$$

$$+\left\{\frac{2[PX+3HT][\cos\sqrt{3}(v_1)t]-2\sqrt{3}\,[(HX)-(TP)](\sin\sqrt{3}(v_1)t)}{X^2+3T^2}\right\} e^{(-A_1-w_1)t} \tag{11}$$

The mean time to system failure MTTF can be obtained from the following relation

$$MTTF = \int_0^\infty R(t)dt=\lim_{t\to\infty}\int_0^t R(t)dt$$

$$MTTF =\lim_{t\to\infty}\int_0^t R(t)dt = \lim_{s\to 0} SL\left\{\int_0^t R(t)dt\right\}= \lim_{s\to 0} S\frac{R^*(S)}{S}$$

$$MTTF=\lim_{s\to 0} R^*(S)$$

$$R^*(S)= L\,(R(t))$$

$$R^*(S) = \frac{m}{(A_1 - w)({A_1}^2+A_1 w + {w_1}^2+3{v_1}^2)S} + \left(\frac{(-A_1+w)^3+A(-A_1+w)^2+B(-A_1+w)+m}{(-A_1+w)(9{w_1}^2+3{v_1}^2)}\right)\frac{1}{S-(-A_1+W)}$$

$$+\frac{2[PX+3HT]}{X^2+3T^2} L\left\{\left(\cos\sqrt{3}(v_1)t\right)e^{(-A_1-w_1)t}\right\} - \frac{2\sqrt{3}\,[(HX)-(TP)]}{X^2+3T^2} L\left\{\left(\sin\sqrt{3}(v_1)t\right)e^{(-A_1-w_1)t}\right\}$$

$$R^*(S) = \frac{m}{(A_1 - w)({A_1}^2+A_1 w + {w_1}^2+3{v_1}^2)S} + \left(\frac{(-A_1+w)^3+A(-A_1+w)^2+B(-A_1+w)+m}{(-A_1+w)(9{w_1}^2+3{v_1}^2)}\right)\frac{1}{S-(-A_1+W)}$$

$$+\frac{2[PX+3HT]}{X^2+3T^2}\frac{S-(-A_1-w_1)}{(S-(-A_1-w_1))^2+(\sqrt{3}(v_1))^2} - \frac{2\sqrt{3}\,[(HX)-(TP)]}{X^2+3T^2}\frac{(\sqrt{3}(v_1))}{(S-(-A_1-w_1))^2+(\sqrt{3}(v_1))^2}$$

$$MTTF=\lim_{s\to 0} R^*(S)$$

As mention above, reliability function has three cases so we find MTTF has following cases:

When $\mu_1=0$ we find

$$MTTF=\lim_{s\to 0} R^*(S)=\frac{m}{(A_1 - w)({A_1}^2+A_1 w + {w_1}^2+3{v_1}^2)S}+\left(\frac{(-A_1+w)^3+A(-A_1+w)^2+B(-A_1+w)+m}{(-A_1+w)(9{w_1}^2+3{v_1}^2)}\right)\frac{1}{S-(-A_1+W)}$$

$$+\frac{2[PX+3HT]}{X^2+3T^2}\frac{S-(-A_1-w_1)}{(S-(-A_1-w_1))^2+(\sqrt{3}(v_1))^2} - \frac{2\sqrt{3}\,[(HX)-(TP)]}{X^2+3T^2}\frac{(\sqrt{3}(v_1))}{(S-(-A_1-w_1))^2+(\sqrt{3}(v_1))^2} \tag{12}$$

$$m = {}_1\ {}_2^2+\mu_1^2\ {}_2+\mu_1\ {}_2\lambda_1 + {}_2^2\lambda_2+\mu_1\mu_2\lambda_2$$

We found that $m \neq 0$ at $\mu_1=0$ **so we cannot find MTTF.**

5. Conclusions

This paper is devoted to deal with a two-unit standby system where each unit has two different modes, (normal & total failure) and two phases of repair. We deal with one system solved by two methods. First method by using Markov-renewal processes and the second method by using Laplace transform techniques to improvement in the system but we find that the method of Laplace transform may solve some systems and may be not because this method depends on the value of (m) must be equal zero and after solving this system we found that $m \neq 0$ so **we cannot find MTTF**

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