Reliability and Availability analysis of two- identical unit cold standby redundant system

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Abstract

This paper deals with two identical unit cold standby redundant system in which each unit has three different modes, (normal, partial failure and total failure) The rate of failure is exponential distribution but the rate of repair is general distribution. The system resolved by supplementary technique and Laplace transforms depends on Complex imaginary roots. Various measures of availability and mean time to failure of the system are studied. Some reliability measures of interest to system designers as well to operation managers have been obtained

Keywords: Availability, Reliability, Cold standby, Laplace transforms, supplementary technique.

1. Introduction

System reliability occupies increasingly more important issues in power plants, manufacturing systems, industrial systems, engineering systems, standby systems, etc. Maintaining a high or required level of reliability is often an essential requirement of the systems. The study of repairable systems is an important component in reliability analysis. Also repairman is one of the essential parts of repairable systems, and can affect the economy of the systems, directly or indirectly. The primary goal of reliability engineering is to improve the performance of a system. In the initial design activity, the redundancy allocation is a direct way of enhancing the reliability of any system. There are two types of redundancy strategies, active and standby. If all the redundant components operate simultaneously from time zero, even though the system needs only one at any given time, such an arrangement is called active redundancy. On the other side there are three variants of standby redundancy such as cold, warm and hot. In the cold standby redundancy, the component does not fail before it operates. In warm standby redundancy, the component is more prone to failure before operation than the cold standby components. In the hot standby redundancy, the failure pattern of component does not depend whether the component is inactive or in operation. The mathematical models for hot standby and active standby arrangements are the same. Lots of work has been done by many researchers in this area. [1] Have studied the behavior of reliability analysis of a system having four components arranged in series. Subsystems A, B, C have single unit whereas subsystem D has three units where one unit is active and the other two are cold standby arranged in parallel. System can completely fail either due to the failure of subsystems A, B and C or due the failure of all units of subsystem D. All failure rates are constant and all repair rates follow the general time distribution. The analysis is carried out using the supplementary variable technique and Laplace transformation for evaluating the reliability measures. [2] two stochastic models for 2(k)-out-of-3(n)redundant system of identical units with repair and inspection are examined stochastically .The system is studied under an operational restriction on the inspection , in case when system has only one unit in operational mode the server has to attend the system for inspection .Semi-Markov processes and Regenerative point technique is adopted to obtain the expressions for measures of system effectiveness such as transition probabilities, mean sojourn times, mean time to system failure, steady state availabilities, [3] In this paper the asymptotic performance of a cold standby system with two identical units is analyzed. When a unit fails it is inspected by a server to check the feasibility of repair or replacement. If repair is not feasible, unit is replaced by new. The server is subjected to failure while performing its job. The system model is developed using semi-Markov approach of Markov processes. The regenerative point technique is used to derive expressions for various measures of system

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performance. The random variables associated with the system (failure time, inspection time, repair time, treatment time) are assumed to follow arbitrary distribution with different probability density functions.[4] deals with a stochastic model of a system consisting of two identical units. Initially, one unit is in operation and other is taken as spare in cold standby. The standby unit may fail after exceeding the maximum redundancy time. The failed standby unit undergoes for inspection immediately to check its possible renewal. If renewal of the unit is not possible, it is replaced by new one. The operative unit goes directly under repair without inspection, after failure. The failure time of unit follows exponential distribution while the inspection and repair times follow arbitrary distributions. The semi-Markov process and regenerative-point technique are used to derive expressions for different reliability and performance indices. [5] Display analysis focuses mainly on coherent systems and series connection of *k*-out-of-*n* standby subsystems with exponentially distributed component lifetimes. primary objective is provide explicit expressions for these performance measures and obtain various characterizations on their mathematical structures. This primarily involves difference of convex functions which are known to be very useful in the context of optimization problems.[6] An *n*-unit system provisioned with a single warm standby is investigated. The individual units are subject to repairable failures, while the entire system is subject to a non repairable failure at some finite but random time in the future. System performance measures for systems observed over a time interval of random duration are introduced. Two models to compute these system performance measures, one employing a policy of block replacement, and the other without a block replacement policy, are developed. Distributional assumptions involving distributions of phase type introduce matrix Laplace transformations into the calculations of the performance measures. It is shown that these measures are easily carried out on a laptop computer using Microsoft Excel. A simple economic model is used to illustrate how the performance measures may be used to determine optimal economic design specifications for the warm standby

This research is devoted to study two identical cold standby where each unit has three different modes, (normal, partial failure and total failure). We also study some special cases to prove that the system, when it is to go to the total failure mode with passing through the partial failure is better than to go to the total failure without passing through the partial failure mode in the non-identical unit or in the identical unit. For example, consider a boiler in a thermal power plant. The boiler can be operated in partial failure mode if the part of the tubing is failed, the part of the system can be blocked and repaired but the system is still operating under partial failure. Any power distribution network is another example of systems with the partial failure mode. The systems resolved by partial differential eq. & Laplace Transform where depend on Complex imagine roots. The mean - time-to-failure, MTTF, and the steady-state availability, A0, for the system is present by graphs. .

1.1. Notations

λ_1 failure rate from O to P.

λ_2 failure rate from P to T.

λ_3 failure rate from O to T.

μ_1 repair rate from P to O.

μ_2 repair rate from T to O

P_i^* (s) Laplace transform of $P_i(t)$

A (t): functions of availability.

R (t): functions of reliability.

MTTF: mean time failure.

Where Laplace transforms (L.T) of $P_i(t)$ is:

$P_i^*(s) = \int_0^\infty e^{-St} P_i(t)$

1.2. Model description

The system is analyzed under the following practical assumptions:

1. Two identical units operate in cold standby.
2. Each unit of the system has three modes, (normal, partial failure and total failure) by the normal mode of the unit; we mean the functioning of the unit with full capacity, by the partial failure mode, functioning with reduced capacity at a specified level, and by total failure mode and functioning with capacity below the specified level.
3. The failure time distributions are assumed to be exponential.

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4. If the repair at the partial mode is completed the unit enter normal mode, otherwise, it may go to the total failure mode.
5. The unit, which is repaired from the total failure state passes into the normal state or standby state.

A single service facility is available to repair a partially failed unit and totally failed unit. The repair facility is not always with the system but can be made available instantaneously whenever needed. The following diagram show system without passing through the partial failure mode.

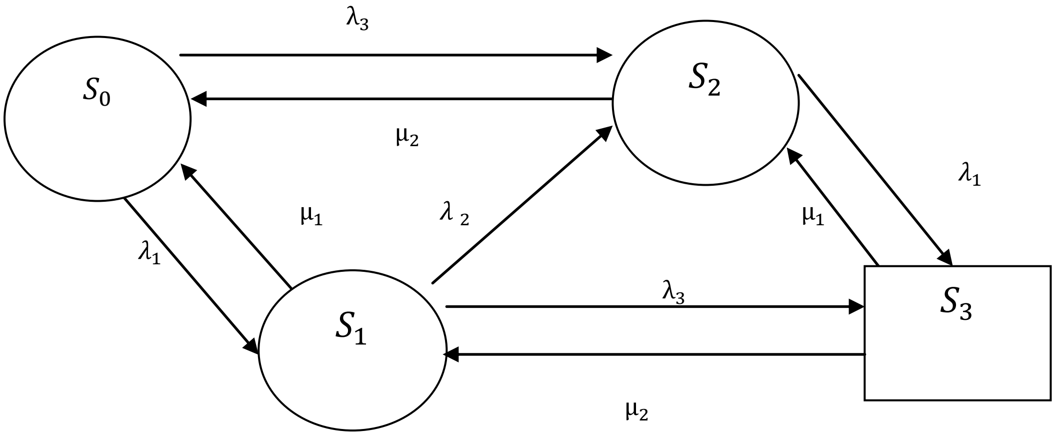

Fig. 1. State transition diagram for two identical units

1.3. Mathematical model description

This part showing the differential equation for the system of fig (1) Transition states

$$\frac{dP_0(t)}{dt} = - [\lambda_1 + \lambda_3]\, P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) \tag{1}$$

$$\frac{dP_1(t)}{dt} = - [\mu_1 + \lambda_2 + \lambda_3] P_1(t) + \lambda_1 P_0(t) + \mu_2 P_3(t) \tag{2}$$

$$\frac{dP_2(t)}{dt} = - [\mu_2 + \lambda_1]\, P_2(t) + \lambda_3 P_0(t) + \mu_1 P_3(t) + \lambda_2 P_1(t) \tag{3}$$

$$\frac{dP_3(t)}{dt} = - [\mu_1 + \mu_2]\, P_3(t) + \lambda_1 P_2(t) + \lambda_3 P_1(t) \tag{4}$$

Initial conditions:

$$P_i\,(0) = \begin{cases} 1 & where\ i = 0 \\ 0 & otherwise \end{cases}$$

Taking Laplace transform of equations (1) – (4), we get

$$[\lambda_1 + \lambda_3 + s]\, P_0^*(s) - \mu_1 P_1^*(s) - \mu_2 P_2^*(s) = P_0\,(0) \tag{5}$$

$$[\mu_1 + \lambda_2 + \lambda_3 + s]\, P_1^*(s) - \lambda_1 P_0^*(s) - \mu_2 P_3^*(s) = P_1\,(0) \tag{6}$$

$$[\lambda_1 + \mu_2 + s]\, P_2^*(s) - \lambda_3 P_0^*(s) - \mu_1 P_3^*(s) - \lambda_2 P_1^*(s) = P_2\,(0) \tag{7}$$

$$[\mu_1 + \mu_2 + s]\, P_3^*(s) - \lambda_1 P_2^*(s) - \lambda_3 P_1^*(s) = P_3\,(0) \tag{8}$$

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Solving equations (5) – (8), by crammer rule, we obtain

$P_0^*(s) = \frac{\Delta 1}{\Delta}$, $P_1^*(s) = \frac{\Delta 2}{\Delta}$

$P_2^*(s) = \frac{\Delta 3}{\Delta}$, $P_3^*(s) = \frac{\Delta 4}{\Delta}$

Where,

$\Delta = s[s^3 + a_1 s^2 + a_2 s + a_3]$, $\Delta_1 = s^3 + As^2 + Bs + m$

$\Delta_2 = \lambda_1 s^2 + (\lambda_1\mu_1 + \lambda_1^2 + 2\mu_2\lambda_1)s + \mu_2\lambda_1^2 + \mu_1\mu_2\lambda_1 + \mu_2\lambda_1\lambda_3 + \lambda_1\mu_2^2$

$\Delta_3 = \lambda_3 s^2 + (\lambda_3\mu_2 + \lambda_3^2 + \lambda_1\lambda_2 + \lambda_2\lambda_3 + 2\mu_1\lambda_3)s +$
$\mu_1\lambda_3^2 + \mu_1\mu_2\lambda_3 + \mu_2\lambda_1\lambda_2 + \lambda_3\mu_1^2 + \mu_1\lambda_3\lambda_2 + \mu_1\lambda_3\lambda_1 + \mu_1\lambda_1\lambda_2 + \mu_2\lambda_3\lambda_2$

$\Delta_4 = \lambda_1^2\lambda_2 + \lambda_3^2\lambda_1 + \lambda_1^2\lambda_3 + \mu_1\lambda_3\lambda_1 + \mu_2\lambda_3\lambda_1 + \lambda_3\lambda_1\lambda_2 + 2\lambda_3\lambda_1 S$

We know that the system contain of (3) up state and (1) down state so

$$P^*(s) = \frac{s^3 + A_1 s^2 + B_1 s + m_1}{s[s^3 + a_1 s^2 + a_2 s + a_3]} \tag{9}$$

Where,

$a_3 = \mu_1^2(\mu_2 + \lambda_3) + \lambda_1^2(\mu_2 + \lambda_3) + \mu_1\mu_2(2\lambda_1 + \lambda_2 + 2\lambda_3) + \lambda_1\lambda_3(2\mu_1 + 2\mu_2) + \lambda_3^2(\mu_1 + \lambda_1) + \mu_2^2(\mu_1 + \lambda_2 + \lambda_1)$

$+\lambda_2\lambda_3(\mu_1 + \mu_2 + \lambda_1) + \lambda_2\lambda_1(\mu_1 + \mu_2 + \lambda_1)$

$a_2 = \mu_1(\mu_1 + 3\mu_2 + \lambda_2 + 2\lambda_1 + 3\lambda_3) + \mu_2(\mu_2 + 2\lambda_3 + 3\lambda_1 + 2\lambda_2) + \lambda_1^2 + 2\lambda_1\lambda_2 + 3\lambda_1\lambda_3 + \lambda_3^2 + \lambda_2\lambda_3$

$a_1 = 2\lambda_1 + \lambda_2 + 2\mu_1 + 2\mu_2 + 2\lambda_3$, $A = 2\mu_1 + 2\mu_2 + \lambda 3 + \lambda 1 + \lambda 2$

$B = 3\mu_1\mu_2 + \mu_1\lambda_1 + \mu_1\lambda_2 + \mu_2\lambda_1 + \mu_1\lambda_3 + \lambda_1\lambda_2 + \lambda_1\lambda_3 + \mu_2^2 + \mu_2\lambda_3 + 2\mu_2\lambda_2$

$m = \mu_1\mu_2^2 + \mu_1^2\mu_2 + \mu_1\mu_2\lambda_1 + \mu_1\mu_2\lambda_2 + \mu_1\mu_2\lambda_3 + \lambda_2\mu_2^2$

$A_1 = 2\mu_1 + 2\mu_2 + 2\lambda_3 + 2\lambda_1 + \lambda_2$

$B_1 = 3\mu_1\mu_2 + 2\mu_1\lambda_1 + \mu_1\lambda_2 + 3\mu_2\lambda_1 + 3\mu_1\lambda_3 + 2\lambda_1\lambda_2 + \lambda_1\lambda_3 + \mu_2^2 + 2\mu_2\lambda_3 + 2\mu_2\lambda_2 + \lambda_1^2 + \lambda_3^2 + \lambda_2\lambda_3$

$m_1 = \mu_1\mu_2^2 + \mu_1^2\mu_2 + 2\mu_1\mu_2\lambda_1 + \mu_1\mu_2\lambda_2 + 2\mu_1\mu_2\lambda_3$

$+\lambda_2\mu_2^2 + \mu_2\lambda_1^2 + \mu_2\lambda_1\lambda_3 + \lambda_1\mu_2^2 + \mu_1\lambda_3^2 + \mu_2\lambda_1\lambda_2 + \lambda_3\mu_1^2 + \mu_1\lambda_3\lambda_2 + \mu_1\lambda_3\lambda_1 + \mu_1\lambda_1\lambda_2 + \mu_2\lambda_3\lambda_2$

2. Cubic equations roots have are 2 cases

After test for D. We found that $D < 0$ so we use second case [All roots are real and unequal]

$$P^*(s) = \frac{s^3 + A_1 s^2 + B_1 S + m_1}{s(s + A_2 - w_0)(S + A_2 - w_2)(S + A_2 - v_2)} \tag{10}$$

Where,

$q = \frac{3a_2 - {a_1}^2}{9}$, $r = \frac{9a_1 a_2 - 2{a_1}^3 - 27a_3}{54}$

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$D = q^3 + r^2$, $u = (r + \sqrt{D})^{\frac{1}{3}}$ $A_2 = \frac{a_1}{3}$, $t = (r - \sqrt{D})^{\frac{1}{3}}$

$w_0 = 2\sqrt{-q}\cos(\frac{\theta}{3})$, $w_2 = 2\sqrt{-q}\cos(\frac{\theta}{3} + 120)$, $v_2 = 2\sqrt{-q}\cos(\frac{\theta}{3} + 240)$, $A_2 = \frac{a_1}{3}$

By using inverse of Laplace Transform of equation, we obtain

$$P(t) = \frac{m_1}{(A_2 - w_0)(A_2{}^2 - A_2 w_2 - A_2 v_2 + w_2 v_2)} + \frac{(-A_2 + w_0)^3 + A_1(-A_2 + w_0)^2 + B_1(-A_2 + w_0) + m_1}{(-A_2 + w_0)(w_0{}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)} e^{(-A_2 + w_0)t}$$

$$+ \frac{(-A_2 + w_2)^3 + A_1(-A_2 + w_2)^2 + B_1(-A_2 + w_2) + m_1}{(-A_2 + w_2)(w_2{}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)} e^{(-A_2 + w_2)t} + \frac{(-A_2 + v_2)^3 + A_1(-A_2 + v_2)^2 + B_1(-A_2 + v_2) + m_1}{(-A_2 + v_2)(v_2{}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)} e^{(-A_2 + v_2)t}$$

2. 2. Availability analysis of the system

We obtain the availability function for this model, from the following relation

$$A(t) = \frac{m_1}{(A_2 - w_0)(A_2{}^2 - A_2 w_2 - A_2 v_2 + w_2 v_2)} + \frac{(-A_2 + w_0)^3 + A_1(-A_2 + w_0)^2 + B_1(-A_2 + w_0) + m_1}{(-A_2 + w_0)(w_0{}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)} e^{(-A_2 + w_0)t}$$

$$+ \frac{(-A_2 + w_2)^3 + A_1(-A_2 + w_2)^2 + B_1(-A_2 + w_2) + m_1}{(-A_2 + w_2)(w_2{}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)} e^{(-A_2 + w_2)t} + \frac{(-A_2 + v_2)^3 + A_1(-A_2 + v_2)^2 + B_1(-A_2 + v_2) + m_1}{(-A_2 + v_2)(v_2{}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)} e^{(-A_2 + v_2)t} \quad (11)$$

The steady-state availability can be obtained from the following relation

$$A = \lim_{t \to \infty} A(t) = \frac{m_1}{(A_2 - w_0)(A_2{}^2 - A_2 w_2 - A_2 v_2 + w_2 v_2)} \quad (12)$$

2.3 .The mean time to failure

The mean time to system failure can be obtained from the following relation

$$MTTF = \int_0^\infty R(t)dt = \lim_{t \to \infty} \int_0^t R(t)dt$$

$$MTTF = -\left(\frac{(-A_2 + w_0)^2 + A_1(-A_2 + w_0) + B_1}{(-A_2 + w_0)(w_0{}^2 - w_0 w_2 - w_0 v_2 + w_2 v_2)}\right) - \left(\frac{(-A_2 + w_2)^2 + A_1(-A_2 + w_2) + B_1}{(-A_2 + w_2)(w_2{}^2 - w_0 w_2 - w_2 v_2 + w_0 v_2)}\right) - \left(\frac{(-A_2 + v_2)^2 + A_1(-A_2 + v_2) + B_1}{(-A_2 + v_2)(v_2{}^2 - w_0 v_2 - w_2 v_2 + w_0 w_2)}\right) \quad (13)$$

The following diagram show system with passing through the partial failure mode

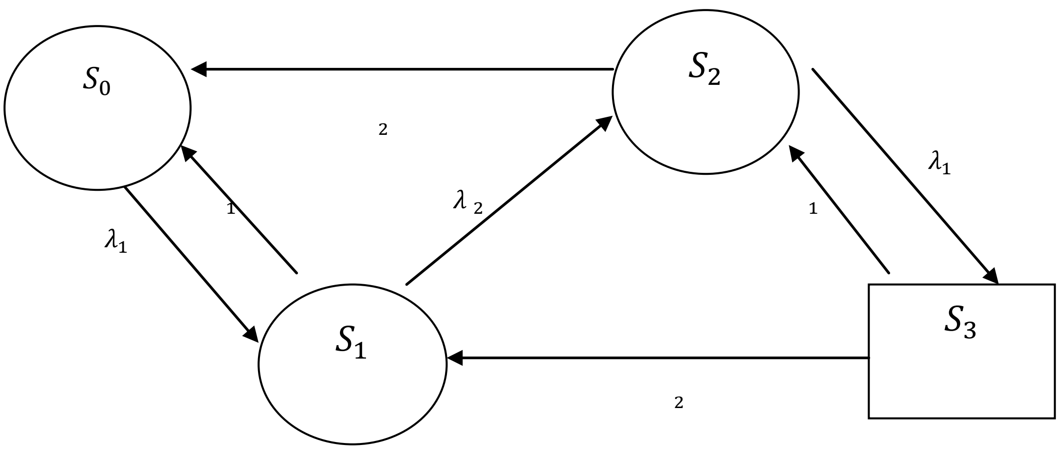

Fig. 2. State transition diagram for two identical units

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3. Mathematical model description

This part showing the differential equation for the system of fig (2) Transition states

$$\frac{dP_0(t)}{dt} = - \; {}_1P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t) \tag{14}$$

$$\frac{dP_1(t)}{dt} = - [\mu_1 + \lambda_2] P_1(t) + \; {}_1P_0(t) + \mu_2 P_3(t) \tag{15}$$

$$\frac{dP_2(t)}{dt} = - [\mu_2 + \lambda_1] P_2(t) + \mu_1 P_3(t) + \; {}_2P_1(t) \tag{16}$$

$$\frac{dP_3(t)}{dt} = - [\mu_1 + \mu_2] P_3(t) + \lambda_1 P_2(t) \tag{17}$$

Initial conditions:

$$P_i(0) = \begin{cases} 1 & where\ i = 0 \\ 0 & otherwise \end{cases}$$

Taking Laplace transform of equations (14) – (17), we get

$$[\lambda_1 + s]\, P_0^*(s) - \mu_1 P_1^*(s) - \mu_2 P_2^*(s) = P_0(0) \tag{18}$$

$$[\mu_1 + \lambda_2 + s]\, P_1^*(s) - \; {}_1P_0^*(s) - \mu_2 P_3^*(s) = P_1(0) \tag{19}$$

$$[\lambda_1 + \mu_2 + s]\, P_2^*(s) - \mu_1 P_3^*(s) - \; {}_2P_1^*(s) = P_2(0) \tag{20}$$

$$[\mu_1 + \mu_2 + s]\, P_3^*(s) - \; {}_1P_2^*(s) = P_3(0) \tag{21}$$

Solving equations (18) – (21), by crammer rule, we obtain

$$P_0^*(s) = \frac{\Delta 1}{\Delta} \quad , \quad P_1^*(s) = \frac{\Delta 2}{\Delta}$$

$$P_2^*(s) = \frac{\Delta 3}{\Delta} \quad , \quad P_3^*(s) = \frac{\Delta 4}{\Delta}$$

Where,

$\Delta = s[s^3 + a_1 s^2 + a_2 s + a_3]$, $\Delta_1 = s^3 + As^2 + Bs + m$

$\Delta_2 = \lambda_1 s^2 + (\lambda_1\mu_1 + \;{}^2_1 + 2\mu_2\;{}_1)\, s + \mu_2\lambda_1^2 + \mu_1\mu_2\;{}_1 + \lambda_1\mu_2^2$

$\Delta_3 = \lambda_1\;{}_2(\mu_1 + \mu_2 + s)$, $\Delta_4 = \lambda_1^2\,\lambda_2$

$a_3 = \mu_2\;{}_1\;{}_2 + \mu_1\;{}_1\;{}_2 + \mu_2\mu_1\;{}_2 + 2\mu_2\mu_1\;{}_1 + \mu_1^2\mu_2 + \mu_2^2\mu_1 + \;{}^2_1\;{}_2 + \;{}^2_1\mu_2 + \mu_2^2\,\lambda_2 + \mu_2^2\,\lambda_1$

$a_2 = 2\lambda_1\;{}_2 + 3\mu_1\mu_2 + 2\mu_1\;{}_1 + \mu_1\;{}_2 + 2\mu_2\;{}_2 + 3\mu_2\;{}_1 + \mu_2^2 + \mu_1^2 + \lambda_1^2$

$a_1 = 2\mu_1 + 2\mu_2 + 2\lambda_1 + \;{}_2$, $A = 2\mu_1 + 2\mu_2 + \lambda_1 + \lambda_2$

$B = \mu_1\;{}_2 + \mu_1\;{}_2 + \lambda_1\mu_2 + 2\lambda_2\mu_2 + 3\mu_1\mu_2 + \mu_2^2 + \mu_1^2 + \;{}_1\;{}_2$

$m = \lambda_1\mu_1\mu_2 + \lambda_2\mu_1\mu_2 + \mu_1^2\mu_2 + \mu_2^2\mu_1 + \mu_2^2\,\lambda_2$

We know that the system contain of (3) up state and (1) down state so

$$P^*(s) = \frac{s^3 + A_1 s^2 + B_1 S + m_1}{s[s^3 + a_1 s^2 + a_2 S + a_3]} \tag{22}$$

$A_1 = 2\mu_1 + 2\mu_2 + 2\lambda_1 + \lambda_2$

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B_1=2μ$_1$ $_1$+μ$_1$ $_2$+3λ_1μ$_2$+ 2λ_2μ$_2$ +3μ$_1$μ$_2$+$μ_2^2$+$μ_1^2$ + $\,_1^2$ + 2λ_1 $_2$

m_1=2λ_1μ$_1$μ$_2$+ $_2$μ$_1$μ$_2$+$μ_1^2$μ$_2$+ $μ_2^2$μ$_1$+μ$_2\lambda_1^2$ + $_1$ $_2$μ$_1$ + $_1$ $_2$μ$_2$+$\lambda_1μ_2^2$+$μ_2^2$ λ_2

4. Cubic equations roots have are two cases

After test for (D) We found that D > 0 so we use First case (D > 0)

$$P^*(s) = \frac{s^3+A_1s^2+B_1s+m_1}{s(s+A_2-W)(s+A_2+w_1-i\sqrt{3}v_1)(s+A_2+w_1+i\sqrt{3}v_1)} \quad (23)$$

By using the inverse of Laplace Transform of equation we obtain

$$P(t) = \frac{m_1}{(A_2-w)(A_2{}^2+A_2w+w_1{}^2+3v_1{}^2)} + \frac{(-A_2+w)^3+A_1(-A_2+w)^2+B_1(-A_2+w)+m_1}{(-A_2+w)\left(9w_1{}^2+3v_1{}^2\right)}e^{(-A_2+W)t}$$

$$+\left\{\frac{2[PX+3HT][\cos\sqrt{3}(v_1)t]-2\sqrt{3}[(HX)-(TP)](\sin\sqrt{3}(v_1)t)}{X^2+3T^2}\right\}e^{(-A_2-w_1)t} \quad (24)$$

Where,

P = (- $A_2{}^3$ - 3$A_2{}^2w_1$ + 9$A_2v_1{}^2$- $w_1{}^3$+ 9$w_1v_1{}^2$- $A_2w_1{}^2$) + A_1 ($A_2{}^2$ + A_2w - 3$v_1{}^2$+ $w_1{}^2$)+ B_1($-A_2 -$ w_1) + m_1

H = (6$A_2v_1w_1$ - 3$v_1{}^3$+ 3$w_1{}^2v_1$ + 3$A_2{}^2v_1$ -A_1A_2v - A_1v_1w + Bv_1)

X = 3v^2w + 6$v_1{}^2A_2$, T = 3wv_1A_2 + 3$w_1{}^2$v − 6$v_1{}^3$

4.1 Reliability System and availability

To obtain the reliability function for this model, we assume that at least one of failed states is absorbing state and the transition rate from this state equal to zero.

4.1.1 System availability

We obtaining the system availability from the relation

$$A(t) = \frac{m_1}{(A_2-w)(A_2{}^2+A_2w+w_1{}^2+3v_1{}^2)} + \frac{(-A_2+w)^3+A_1(-A_2+w)^2+B_1(-A_2+w)+m_1}{(-A_2+w)\left(9w_1{}^2+3v_1{}^2\right)}e^{(-A_2+W)t}$$

$$+\left\{\frac{2[PX+3HT][\cos\sqrt{3}(v_1)t]-2\sqrt{3}[(HX)-(TP)](\sin\sqrt{3}(v_1)t)}{X^2+3T^2}\right\}e^{(-A_2-w_1)t} \quad (25)$$

So the steady – state availability (A) from the following relation

$$A = \lim_{t\to\infty} A(t) = \frac{m_1}{(A_2-w)(A_2{}^2+A_2w+w_1{}^2+3v_1{}^2)} \quad (26)$$

4.1.2. System reliability

supposing that at least one for failed states is absorbing state and the state transition rate equal to zero, so the reliability function for this model as like that

$$R(t) = \frac{m_1}{(A_2-w)(A_2{}^2+A_2w+w_1{}^2+3v_1{}^2)} + \frac{(-A_2+w)^3+A_1(-A_2+w)^2+B_1(-A_2+w)+m_1}{(-A_2+w)\left(9w_1{}^2+3v_1{}^2\right)}e^{(-A_2+W)t}$$

$$+\left\{\frac{2[PX+3HT][\cos\sqrt{3}(v_1)t]-2\sqrt{3}[(HX)-(TP)](\sin\sqrt{3}(v_1)t)}{X^2+3T^2}\right\}e^{(-A_2-w_1)t} \quad (27)$$

The mean time to system failure can be obtained from the following relation

$\text{MTTF} = \int_0^{\infty} R(\mathrm{t})\mathrm{dt} = \lim_{t\to\infty} \int_0^{t} R(\mathrm{t})\mathrm{dt}$

$$\text{MTTF} = -\left(\frac{(-A_2+w)^2+A_1(-A_2+w)+B_1}{(-A_2+w)(9w_1{}^2+3v_1{}^2)}\right) - \frac{(2[PX+3HT])(-A_2-w_1)+6v_1[HX-TP]}{(X^2+3T^2)[(-A_2-w_1)^2+3v_1{}^2]} \quad (28)$$

5. Study behavior of the system through graphs

To observe the system behavior, we plot the MTTF and the steady-state availability for the models, against α (failure rate from O to P) keeping the other parameters fixed.. we setting

$_2 = 0.25$, $\mu_2=0.3$, $\mu_1=0.1$ $\lambda_3=0.5$, $\lambda_1= 0.2, 0.4, 0.6, 0.8, 1, 1.2$

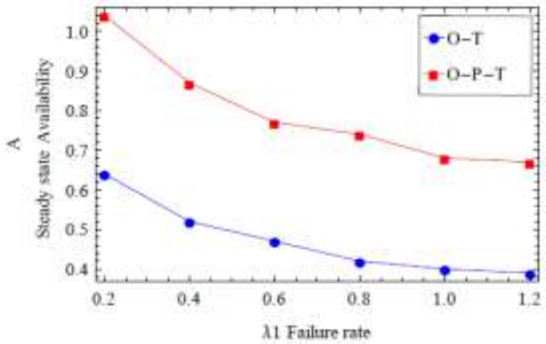

Fig 3 the Steady state Availability w.r.t. Failure Rate

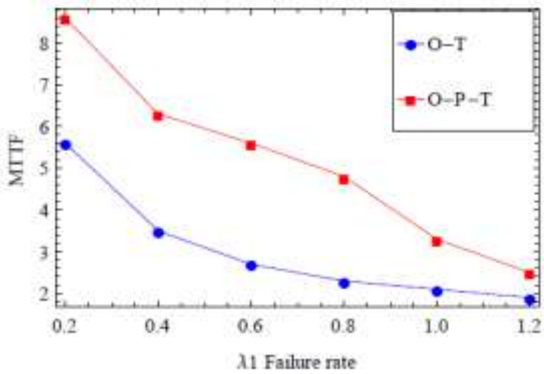

Fig 4 the mean time failure w.r.t. Failure Rate

6. Conclusions

The combination of standby and maintainability is of great importance in many real time systems operating in the machining environment. In this investigation, we have addressed the issue of improvement in the system, when it is to go to the total failure mode with passing through the partial failure is better than to go to the total failure without passing through the partial failure mode in the non-identical unit or in the identical unit by supplementary technique and Laplace transforms depends on Complex imaginary roots. Various measures of availability and (MTTF) of the system studied, some reliability measures of interest to system designers as well to operation managers have been obtained. We find that the system which passing through the partial failure mode is better than the system which without passing through the partial failure mode

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